



## Definition of Multi-State Weighted k-out-of-n: F System

Y. Ding, Enrico Zio, Yan-Fu Li, Lin Cheng, Qiu-Wei Wu

### ► To cite this version:

Y. Ding, Enrico Zio, Yan-Fu Li, Lin Cheng, Qiu-Wei Wu. Definition of Multi-State Weighted k-out-of-n: F System. International Journal of Performability Engineering, 2012, 8 (2), pp.217-219. hal-00704898

**HAL Id: hal-00704898**

**<https://hal-centralesupelec.archives-ouvertes.fr/hal-00704898>**

Submitted on 6 Jun 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## Definition of Multi-state Weighted k-out-of-n: F System

YI DING<sup>a\*</sup>, ENRICO ZIO<sup>b,c</sup>, YANFU LI<sup>c</sup>, LIN CHENG<sup>d</sup>, and QIUWEI WU<sup>a</sup>

<sup>a</sup>Department of Electrical Engineering, Technical University of Denmark, Denmark.

<sup>b</sup>Ecole Centrale Paris - Supelec, Paris, France.

<sup>c</sup>Politecnico di Milano, Milan, Italy, Dipartimento di Energia.

<sup>d</sup>Department of Electrical Engineering, Tsinghua University, China.

(Received on month date, year)

**Abstract** – The Multi-state Weighted k-out-of-n System model is the generalization of the Multi-state k-out-of-n System model, which finds wide applications in industry. However only Multi-state Weighted k-out-of-n: G System models have been defined and studied in most recent research works. The mirror image of the Multi-state Weighted k-out-of-n: G System – the Multi-state Weighted k-out-of-n: F System has not been clearly defined and discussed. In this short communication, the basic definition of the Multi-state Weighted k-out-of-n: F System model is proposed. The relationship between the Multi-state Weighted k-out-of-n: G System and the Multi-state Weighted k-out-of-n: F System is also analyzed.

**Keywords:** multi-state system, k-out-of-n system, weighted k-out-of-n system

### Notation:

$n$	number of components in the system
$W$	weight of the Multi-state Weighted k-out-of-n: F System
$w_i$	random variable representing the weight of component $i$
$w_{ij}$	weight of component $i$ in state $j$
$G$	weight of the Multi-state Weighted k-out-of-n: G System
$M$	perfect performing state of the system
$\Phi^F$	state of the system
$k_j^F$	a predefined value corresponding state $j^F$
$\Omega_\phi$	composition operator of UGF

### 1. Introduction

A Multi-state Weighted k-out-of-n: G System model is a flexible tool for modeling complex engineering systems [1]. However, the computational burden becomes the crucial factor for reliability evaluation of Multi-state Weighted k-out-of-n: G System [2]. The methods for reliability evaluation can be computationally inefficient, which are limited to analysis of small systems. The Multi-state Weighted k-out-of-n: F System is an equivalent of the Multi-state Weighted k-out-of-n: G System [3]. A Multi-state Weighted k-out-of-n: G System can be transformed into the equivalent of the Multi-state Weighted k-out-of-n: F system and then be analyzed using the reliability evaluation methods. It has been illustrated in [3] that this transformation can be used for analyzing reliabilities of large-scale Multi-state k-out-of-n Systems. Therefore reliability evaluation of Multi-state

---

\*Corresponding author's email: [yding@elektro.dtu.dk](mailto:yding@elektro.dtu.dk)

Weighted  $k$ -out-of- $n$ : F System may provide an efficient way for reducing computational complexities.

However the Multi-state Weighted  $k$ -out-of- $n$ : F System has not been clearly defined and studied, which exists in real life applications. Electric power generating system can be seen as a typical example of Multi-state Weighted  $k$ -out-of- $n$ : F System: The demand of the power generating system has various levels, characterized by different states. If the available capacity of generating system is lower than the demand, the system will fail.

In this short communication, the basic model of the Multi-state Weighted  $k$ -out-of- $n$ : F System is defined in Section II. The relationship between the Multi-state Weighted  $k$ -out-of- $n$ : G System and the Multi-state Weighted  $k$ -out-of- $n$ : F System is also analyzed.

## 2. Multi-state Weighted $k$ -out-of- $n$ : F system models

In a Multi-state Weighted  $k$ -out-of- $n$  System, component  $i$  in state  $j$  carries a weight of  $w_{ij}$ ,  $1 \leq i \leq n$ ,  $0 \leq j \leq M$ , representing a given contribution to system performance [1]. The formal definition of the Multi-state Weighted  $k$ -out-of- $n$ :F System model is given below.

**Definition of a Multi-state Weighted  $k$ -out-of- $n$ : F System:** Each component in the system can be in  $M + 1$  possible states:  $0, 1, 2, \dots, M$ , with  $M$  being the perfect state of the system. The system is in state  $j^F$  or lower ( $1 \leq j^F \leq M$ ) if the total weight of all components is lower than or equal to a predefined value  $k_j^F$  ( $k_1^F < k_2^F < \dots < k_M^F$ ). Denoting by  $\Phi^F$  the state of the system, we then have  $\Pr(\Phi^F \leq j^F) = \Pr(W \leq k_j^F)$ , where

$$W = \sum_{i=1}^n w_i. \text{ Because state } M \text{ is the perfect state of the system, we have } \Pr(\Phi^F \leq M) = 1.$$

The defined Multi-state Weighted  $k$ -out-of- $n$ : F System can be considered as a mirror image of the Multi-state Weighted  $k$ -out-of- $n$ : G System:

Let  $G = \sum_{i=1}^n w_{iM} - W$ . If  $W \leq k_j^F$ , then  $G \geq \sum_{i=1}^n w_{iM} - k_j^F$ . Let  $k_j^G = \sum_{i=1}^n w_{iM} - k_j^F$  represent the predefined value for the state  $j^G$  ( $j^G = M + 1 - j^F$ ,  $1 \leq j^G \leq M$ ), then we have an order  $k_1^G < k_2^G < \dots < k_M^G$ . Suppose the system is in state  $j^G$  or above if  $G$  is greater than or equal to  $k_j^G$ . We then have  $\Pr(\Phi^G \geq j^G) = \Pr(G \geq k_j^G)$ . Based on the definition [1], the system can be represented as a Multi-state Weighted  $k$ -out-of- $n$ : G System. We have  $\Pr(\Phi^G \geq j^G) = \Pr(\Phi^F \leq j^F)$ , which indicates that the state distribution of a Multi-state Weighted  $k$ -out-of- $n$ : F System is equal to the state distribution of a Multi-state Weighted  $k$ -out-of- $n$ : G System.

## 3. Example

Consider a multi-state weighted  $k$ -out-of- $n$ : F system with two components. Each component, and the system, have three possible states: 0, 1, and 2. Table 1 gives the weight and probability parameters of these components. To obtain the state distribution of the system, we model it by the universal generating function (UGF) method, a sound approach for reliability evaluation of multi-state systems. UGF was introduced in [4] to reduce the computational complexity for MSS. Further developments and applications of the UGF were presented in [5] - [7]. In this example,  $n=2$ ,  $M=2$ ,  $k_1^F = 2.2$ , and  $k_2^F = 4.1$ . Based on the individual UGFs of the original components, the UGF of the system can be

obtained by using the  $\Omega_\phi$  operator.

TABLE 1  
PARAMETERS OF THE COMPONENTS

State $j^F$ :	0	1	2
$w_{1j}^F$	0	1	2.1
$w_{2j}^F$	0	1.2	2
$p_{1j}^F$	0.1	0.4	0.5
$p_{2j}^F$	0.1	0.2	0.7
$k_j^F$	0	2.2	4.1

$$\begin{aligned}
 U_{sys}(z) &= \Omega_\phi\{U_1(z), U_2(z)\} \\
 &= \Omega_\phi\{(0.1 \cdot z^0 + 0.4 \cdot z^1 + 0.5 \cdot z^{2.1}), (0.1 \cdot z^0 + 0.2 \cdot z^{1.2} + 0.7 \cdot z^2)\} \\
 &= 0.01 \cdot z^0 + 0.04 \cdot z^1 + 0.02 \cdot z^{1.2} + 0.07 \cdot z^2 + 0.05 \cdot z^{2.1} + 0.08 \cdot z^{2.2} + 0.28 \cdot z^3 + 0.1 \cdot z^{3.3} + 0.35 \cdot z^{4.1}
 \end{aligned}
 \tag{1}$$

The above UGF gives the probability distribution of performance (performance distribution) of the system; from it, we can obtain the system state distribution for values

$k_1^F = 2.2$ , and  $k_2^F = 4.1$  :

$$\Pr(\Phi^F \leq 2) = \Pr(W \leq 4.1) = 0.01 + 0.04 + 0.02 + 0.07 + 0.05 + 0.08 + 0.28 + 0.1 + 0.35 = 1$$

$$\Pr(\Phi^F \leq 1) = \Pr(W \leq 2.2) = 0.01 + 0.04 + 0.02 + 0.07 + 0.05 + 0.08 = 0.27$$

Therefore the probabilities for the system to be in state 2 or below, and in state1 or below are 1 and 0.27, respectively.

#### 4. Conclusions

In this short communication, we define the Multi-state Weighted k-out-of-n: F System model. More comprehensive studies on its use and related implications are being conducted and will be presented in a full paper.

#### References

1. Li, W., and M. J. Zuo. *Reliability evaluation of multi-state weighted k-out-of-n systems*. Reliability Engineering and System Safety 2008; 93(1): 160-167.
2. Ding Y., M. J. Zuo, A. Lisnianski and W. Li. *A Framework for Reliability Approximation of Multi-State Weighted k-out-of-n Systems*. IEEE Trans. on Reliability 2010; 59(2): 297-308.
3. Zuo M. J., and Z. G. Tian. *Performance Evaluation of Generalized Multi-State k-out-of-n systems*. IEEE Trans. on Reliability 2006; 55(2): 319-327.
4. Ushakov I. *Universal generating function*. Soviet J. Computer System Science 1986; 24(5): 118-129.
5. Lisnianski A., and G. Levitin. *Multi-State System Reliability Assessment, Optimization, Applications*. World Scientific, 2003.
6. Levitin G., *Universal Generating Function and Its Applications*. Springer, 2005.
7. Ding Y., and A. Lisnianski. *Fuzzy Universal Generating Functions for Multi-state System Reliability Assessment*. Fuzzy Sets & Systems 2008; 159 (3): 307-324.

#### Acknowledgment

This research was partially supported by State Key Lab. of Power System, Tsinghua University, Beijing 100084, China (project number: SKLD10KM03).